On the Extraction of the Neutron Spin Structure Functions and the Gerasimov – Drell – Hearn Integral from $^3\vec{H}e(\vec{e},e')X$ data

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Abstract

Nuclear effects in polarized inelastic electron scattering off polarized ^3He are discussed; in the kinematics of future experiments at CEBAF, Fermi motion effects are found to be much larger than in deep inelastic scattering. It is shown that improperly describing nuclear dynamics would lead to the extraction of unreliable neutron spin structure functions. On the other hand side, a simple and workable equation relating the Gerasimov – Drell – Hearn Integral for the neutron to the corresponding quantity for ^3He is proposed.

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The measurement of the polarized nucleon Spin Structure Functions (SSF) g_1 and g_2 in the resonance region allows one to check the helicity structure of the photon – nucleon coupling between the Deep Inelastic Scattering (DIS) region and the real photon limit [1]. Recently it has been proposed at CEBAF to study the SSF in a wide range of energy $(0.2\ GeV \leq \nu \leq 3\ GeV)$ and momentum $(0.15\ GeV^2 \leq Q^2 \leq 2\ GeV^2)$ transfers, for both the proton [2] and the neutron, using in the latter case polarized deuteron [3] and ^3He [4] targets.

For any spin $\frac{1}{2}$ hadronic target A, the SSF g_1^A reads as follows [5]

$$g_1^A(\nu, Q^2) = \frac{M_A K}{8\pi^2\alpha(1+\frac{Q^2}{\nu^2})}\left[\sigma_{1/2}(\nu, Q^2) - \sigma_{3/2}(\nu, Q^2) + \frac{2\sqrt{Q^2}}{\nu}\sigma_{TL}(\nu, Q^2)\right] \qquad (1)$$

where $\sigma_{1/2(3/2)}(\nu, Q^2)$ is the cross section for photon – hadron scattering with total helicity $1/2\ (3/2)$, $\sigma_{TL}(\nu, Q^2)$ is the interference between the transverse and the longitudinal cross sections, K is the photon flux and M_A is the hadron mass.

A relevant quantity related to the SSF g_1^N of the nucleon is the Gerasimov – Drell – Hearn (GDH) Integral:

$$I_{GDH}(Q^2) = \int\limits_{\nu_{th}}^{\infty} \frac{d\nu}{\nu}\left(\sigma_{1/2}(\nu, Q^2) - \sigma_{3/2}(\nu, Q^2)\right), \qquad (2)$$

where $\nu_{th} = (Q^2 + 2m_\pi M + m_\pi^2)/2M$ is the threshold energy for the pion-electroproduction, m_π is the pion mass and M is the nucleon mass. The integral I_{GDH} gives in the real photon limit the GDH Sum Rule [6]:

$$\begin{aligned}
I_{GDH}(Q^2 = 0) &= \int\limits_{\nu_{th}}^{\infty} \frac{d\nu}{\nu}\left(\sigma_{1/2}(\nu, Q^2 = 0) - \sigma_{3/2}(\nu, Q^2 = 0)\right) = -\frac{2\pi^2\alpha}{M^2}\kappa^2 \\
&\simeq \begin{cases} -0.53 & GeV^{-2} \quad \text{for protons} \\ -0.60 & GeV^{-2} \quad \text{for neutrons} \end{cases}
\end{aligned} \qquad (3)$$

where κ is the anomalous magnetic moment of the nucleon.

Using (1) and (2), $I_{GDH}(Q^2)$ can be related in the large Q^2 limit to the first moment of the SSF g_1 and, by using the EMC result for the DIS SSF of the proton g_1 [7], one gets, around $Q^2 \simeq 10\ GeV^2$, $I_{GDH}(Q^2) \simeq 0.14/Q^2$, which is evidence of large changes in the helicity structure of the γp coupling between the real photon limit and the DIS region, leading to a change of sign of $I_{GDH}(Q^2)$ at some value of Q^2. Much theoretical work has

been produced, both in the real photon limit [8] and at finite values of Q^2 [9], in order to understand this behavior, which should be mainly due to the electroexcitation of nucleon resonances.

Extensive experimental programs are planned to investigate not only the Q^2 evolution of the GDH Integral, but also, for the first time, the proton and neutron GDH Sum Rule [10]; in the latter case, several analyses of the available unpolarized photoproduction data do not agree with the prediction (3) for the neutron, while similar estimates seem to give the correct GDH value for the proton; it is for this reason that the neutron measurement is of particular relevance [1].

It is worth noticing that the measurement of the neutron SSF in the CEBAF experiments will also contribute to the investigation of the low Q^2 evolution of the Bjorken sum rule, which requires an accurate knowledge of g_1^n for $0 \leq x \leq 1$, being $x = Q^2/2M\nu$ the Bjorken variable.

Nuclear effects on the SSF g_1 of the deuteron have already been studied [11] in the kinematics of the planned CEBAF experiment; the aim of this letter is to present the results of calculations of nuclear effects on the spin structure function of ^3He.

A convolution model for the nuclear SSF $g_{1(2)}^A$ for any spin $\frac{1}{2}$ nucleus has been obtained in [12, 13]; it reads as follows

$$
\begin{aligned}
g_1^A\left(Q^2, \nu\right) = & \sum_{N=p,n} \int dz \int dE \int d\vec{p}\, \frac{M}{E_p} \frac{M\nu}{p \cdot q}\, g_1^N\left(z, \nu, Q^2\right) \Big\{ P_{\parallel}^N(\vec{p}, E) \\
& + T^N(\vec{p}, E, Q^2) \Big\} \delta\left(z + \frac{M^2 - p \cdot p}{2M\nu} - \frac{q \cdot p}{M\nu}\right)
\end{aligned}
\tag{4}
$$

where E is the removal energy, $p \equiv (M_A - \sqrt{(E + M_A - M)^2 + |\vec{p}|^2},\ \vec{p})$ is the $4-$momentum of the bound nucleon, $E_p = \sqrt{M^2 + |\vec{p}|^2}$ and $g_1^N(z, \nu, Q^2)$ is the nucleon SSF. Nuclear effects are described by the quantities $P_{\parallel}^N(\vec{p}, E)$ and $T^N(\vec{p}, E, Q^2)$, which are both related to the elements of the 2x2 matrix, representing the spin dependent spectral function of a nucleon inside a nucleus with polarization $\vec{S}_A$ [14]. The elements of this matrix are

$$
P_{\sigma,\sigma',\mathcal{M}}^N(\vec{p}, E) = \sum_{f_{A-1}} \langle \vec{p}, \sigma; \psi_{A-1}^f | \psi_{J\mathcal{M}} \rangle\, \langle \psi_{J\mathcal{M}} | \psi_{A-1}^f; \vec{p}, \sigma' \rangle\, \delta(E - E_{A-1}^f + E_A)
\tag{5}
$$

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where $|\psi_{JM}\rangle$ is the ground state of the target nucleus polarized along $\vec{S}_A$, $|\psi^f_{A-1}\rangle$ is an eigenstate of the (A-1) nucleon system corresponding to an energy E^f_{A-1}, $|\vec{p}, \sigma\rangle$ is the plane wave for the nucleon $N \equiv p(n)$. In particular, $P^N_{\parallel}(\vec{p}, E) = P_{\frac{1}{2}\frac{1}{2},\frac{1}{2}}(\vec{p}, E) - P_{-\frac{1}{2}-\frac{1}{2},\frac{1}{2}}(\vec{p}, E)$, whereas the Q^2 dependent term $T^N(\vec{p}, E, Q^2)$, which gets contribution from both g^N_1 and g^N_2, depends also upon $P_{\frac{1}{2}-\frac{1}{2},\frac{1}{2}}(\vec{p}, E)$; this term has been found to give a very small contribution in both the Deep Inelastic Scattering [12] as well as in the present calculation of the resonance region: for this reason it will be omitted hereafter. In the DIS kinematics $(Q^2 \to \infty, \nu \to \infty)$, Eq. (4) gives the simple convolution formula described in Ref [12]:

$$g^A_1(x) = \sum_N \int_x^A dz \, \frac{1}{z} \, g^N_1\left(\frac{x}{z}\right) \, G^N(z) \ , \tag{6}$$

with the light – cone momentum distribution given by

$$G^N(z) = \int dE \int d\vec{p} \, P^N_{\parallel}(\vec{p}, E) \delta\left(z - \frac{p^+}{M}\right) \tag{7}$$

where $p^+ = p^0 - \vec{p} \cdot \vec{q}/|\vec{q}|$ is the light – cone momentum component.

It is well known [15] that the impulse approximation used above is at variance by about 4 % with the experimental results on β decay, which are related to the integral of the difference of the SSF of ^3H, g^{3H}_1, and ^3He, g^{3He}_1, through the Bjorken Sum Rule for the three nucleon systems. Recently [16] it has been argued that nuclear shadowing at $x \le 0.05$ may be responsible for such a disagreement, and could influence g^{3He}_1 up to $x \simeq 0.15$ This effect should not affect our results, which involve larger x values in the kinematics of the planned experiments, at least for $Q^2 \ge 0.3 \ GeV^2$.

In the present letter the ^3He structure function g^{3He}_1 has been calculated in the resonance region ($W_{th} = M + m_\pi < \tilde{W} < 2 \ GeV$, being $\tilde{W} = \sqrt{(p+q)^2}$ the invariant mass of the virtual photon – bound nucleon system), by evaluating the following equation

$$g^A_1\left(x, Q^2\right) = \sum_{N=p,n} \int_{W_{th}} d\tilde{W} \, \tilde{W} \int dE \int d\vec{p} \, \frac{M}{E_p E_X} \frac{M\nu}{p \cdot q} \, g^N_1\left(x, Q^2, \tilde{W}\right)$$
$$P^N_{\parallel}(\vec{p}, E) \delta\left(\nu + M_3 - E_R - E_X\right) \tag{8}$$

obtained from Eq. (4) by changing the integration variable z into $\tilde{W} = \sqrt{(p+q)^2} = \sqrt{2M\nu z + M^2 - Q^2}$, being $E_X = \sqrt{\tilde{W}^2 + (\vec{p} + \vec{q})^2}$ the energy of the hadronic state pro-

duced by the interaction of the struck nucleon with the incoming virtual photon, and $E_R = \sqrt{(E + M_A - M)^2 + |\vec{p}|^2}$ that of the recoiling $2-$ body system.

The model of [9] for the transverse virtual photon absorption cross sections $\sigma_{1/2}$ and $\sigma_{3/2}$ has been used to obtain the nucleon SSF g_1^N according to Eqs. (1). In this model, the contribution of the resonance R is written in the following way:

$$\sigma^R_{1/2(3/2)}(\nu, Q^2) = \frac{4M}{W_0 \Gamma_0} A^{R\ 2}_{1/2(3/2)}(Q^2)\, B(\nu, Q^2) \tag{9}$$

where M and W_0 are the nucleon and resonance masses, Γ_0 is the total width of the resonance, $B(\nu, Q^2)$ is the extension to electroproduction of the Breit – Wigner param- eterization given in [17] for photoproduction, $A^R_{1/2(3/2)}(Q^2)$ is the helicity amplitude for the excitations of the resonances. In this model, the amplitudes pertaining to the res- onant states $P_{33}(1232)$, $D_{13}(1520)$, $S_{11}(1535)$ and $F_{15}(1680)$ have been parametrized by using the existing experimental data, while other states have been added by using the predictions of the Single Quark Transition Model; a non resonant background due to the single pion Born term has also been included in the calculations. In the kinematics of the planned experiments the longitudinal asymmetry can be approximated by the relation: $A^{^3He}(x, Q^2) \simeq 2xg_1^{^3He}(x, Q^2)/F_2^{^3He}(x, Q^2)$ where $F_2^{^3He}(x, Q^2)$ is the unpolarized structure function of ^3He.

Calculations have been performed in the resonance region for both $g_1^{^3He}$ and $F_2^{^3He}$, using for the former eq. (8) and for the latter the convolution approach described in [18]. The results are presented in Figs. 1 and 2, where they are compared with those obtained in DIS. In the latter case, as it can be seen from Fig. 1 (a), the following equation has been shown [12] to be a reliable approximation of the convolution formula, at least for $x \leq 0.8$:

$$g_1^{^3He}(x, Q^2) \approx 2p_p g_1^p(x, Q^2) + p_n g_1^n(x, Q^2) \tag{10}$$

where $p_{p(n)}$ is the effective nucleon polarization, produced by the S' and D waves in the ground state of ^{3}He ($p_n = 1 - \frac{2}{3}P(S') - \frac{4}{3}P(D)$, $p_p = -\frac{1}{3}(P(D) - P(S'))$, being $P(D)$ and $P(S')$ the D- and S'- wave probabilities, and given by

$$p_N = \int dE \int d\vec{p}\, P_{\parallel}^N(\vec{p}, E)\ . \tag{11}$$

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It can be seen from Fig. 1 (b) that an approximation similar to Eq. (10) does not hold in the resonance region, which means that nuclear effects are relevant; we found that binding effects are very small, so that it is the Fermi motion which is responsible for the relevant broadening and damping of the peaks associated to the excitation of the most prominent resonant states in the nuclear medium. This fact is a well known result also in the unpolarized case, as it can be seen in Fig. 2. Fig. 2 (a) shows that, in DIS kinematics, the unpolarized structure function $F_2^{^3He}$, calculated by taking into account Fermi motion and binding, hardly differs from the sum of the structure functions of the free nucleons, whereas Fig. 2 (b) shows that in the resonance region ($Q^2 = 1\ GeV^2$) the resonant peaks are strongly damped by Fermi motion. On the other hand side, we found that the proton contribution to the nuclear g_1 is small, both in the DIS and resonance regions, so that in both regions $^3\vec{H}e$ is a nice effective polarized neutron target.

In DIS, the quantity

$$\tilde{g}_1^n(x, Q^2) = \frac{1}{p_n} \left[g_1^{^3He}(x, Q^2) - 2 p_p g_1^p(x, Q^2) \right] \ , \tag{12}$$

obtained by inverting Eq. (10) and calculated using the convolution formula for $g_1^{^3He}(x, Q^2)$, differs from the free neutron SSF g_1^n, by at most 4%. Such a small difference is rather independent of the form of any well behaved g_1^N, and therefore Eq. (10) can be considered a workable formula for extracting $g_1^n(x, Q^2)$ from the experimental $g_1^{^3He}(x, Q^2)$. The same doesn't hold in the resonance region, so that an alternative way for such an extraction has to be figured out. A possible solution is to use the method described in [19], initially applied in the deep inelastic region and extended in Ref. [11] to the extraction of the neutron spin dependent structure functions at finite Q^2 from the deuteron data.

We would like to stress that in this process the $P_{33}(1232)$ (Δ) resonance, which gives a relevant contribution to the nuclear SSF, is produced with a large average 3 – momentum $|\vec{p}_\Delta|$. For example, at $Q^2 \simeq 1\ GeV^2$, $|\vec{p}_\Delta| \simeq 6\ f^{-1}$ and it remains large even at $Q^2 \simeq 0.5\ GeV^2$ ($|\vec{p}_\Delta| \simeq 4.5\ f^{-1}$). Moreover, at any value of Q^2, heavier resonances are excited with a 3 – momentum larger than $|\vec{p}_\Delta|$. It means that the produced resonant states come out from the nucleus carrying high kinetic energy with respect to the two – body spectator system; thus the final state interactions, which have been disregarded in the

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present impulse approximation approach, should not play a relevant role.

Finally, let us discuss the role of nuclear effects on the GDH Integral. In the DIS limit, it has been observed [12] that the integrals of the formula (4) and that of the approximation Eq. (10) differ by a negligible amount. Then the integral Γ^n of g_1^n ($\Gamma^n = \int g_1^n(x)\, dx$) can be easily obtained from the experimentally known Γ^{3He} and Γ^p integrals, by integrating Eq. (12), obtaining

$$\tilde{\Gamma}^n(Q^2) = \frac{1}{p_n}\left[\Gamma^{3\mathrm{He}}(Q^2) - 2p_p\Gamma^p(Q^2)\right] \ . \tag{13}$$

Let us check whether such a procedure can be applied to the resonance region. To this end let us introduce, for any spin $\frac{1}{2}$ target A, the following integral

$$I^A(Q^2) = \frac{8\pi^2\alpha}{M}\int_{\nu_{th}}^{\infty}\frac{d\nu}{\nu}\frac{\left(1+\frac{Q^2}{\nu^2}\right)}{K}g_1^A(\nu, Q^2) \ , \tag{14}$$

which, in the case of the nucleon, coincides with the GDH integral (2), provided the interference contribution is disregarded in the definition of $g_1^A(x, Q^2)$ (Eq. (1)), which will be assumed hereafter. In Figure 3 we show the quantity

$$\tilde{I}^n(Q^2) = \frac{1}{p_n}\left[I^{3\mathrm{He}}(Q^2) - 2p_p I^p(Q^2)\right] \ , \tag{15}$$

calculated using in (14) the model described in [9] to evaluate g_1^p and the convolution formula (8) to obtain $g_1^{3\mathrm{He}}$. It can be seen that this curve differs at most by 5% from the free neutron $I^n(Q^2)$, obtained using in (14) the model given in [9] for g_1^n. It appears therefore that the simple formula (15) could be used to get the neutron GDH Integral from the measured $I^{3He}(Q^2)$ integral. This quantity is also shown in the Figure: a comparison with the integral corresponding to the free neutron, shows that nuclear structure effects are large, but can be safely taken care of by simply using the effective polarizations.

To sum up, our calculations show that in the resonance region $g_1^{3He}(x, Q^2) \neq p_n g_1^n(x, Q^2)$ $+2p_p g_1^p(x, Q^2)$, but the integrals of these two quantities are very similar. Calculations with different types of g_1^N would be very useful in clarifying the model dependence of such a result.

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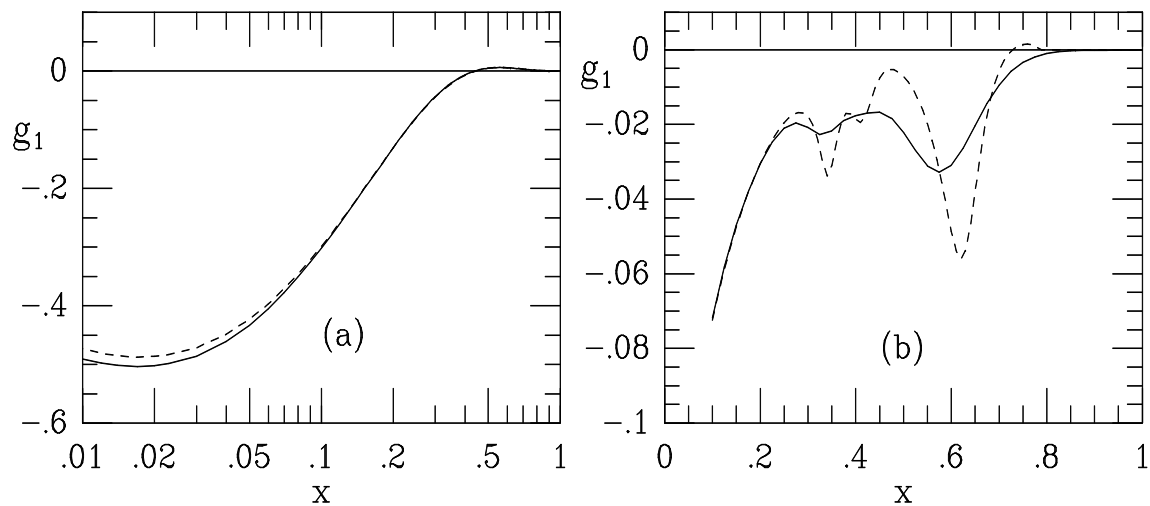[12] C. Ciofi degli Atti, S. Scopetta, E. Pace, and G. Salmè, Phys. Rev. C 48 (1993) R968.

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Captions

Figure 1: $g_1^{^3He}$ in DIS ($Q^2 = 10\,GeV^2$) (a) [12], and in resonance ($Q^2 = 1\;GeV^2$) (b) regions, obtained by considering Fermi motion and binding (full). The dashed curve represents the same functions obtained considering the proton and neutron effective polarization in ^3He as the only relevant nuclear effects (Eq. (10)).

Figure 2: $F_2^{^3He}$ in DIS ($Q^2 = 10\,GeV^2$) (a) and in resonance ($Q^2 = 1\,GeV^2$) (b) regions, calculated by properly considering Fermi motion and binding (full curves) and by summing the unpolarized structure functions of the nucleons (dotted curves).

Figure 3: The integral $\tilde{I}^n(Q^2)$, Eq. (15) (crosses), compared with $I^n(Q^2)$ [9] (full) and with $I^{^3He}(Q^2)$, Eq. (14) (dots).

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FIGURE 1

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FIGURE 2

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FIGURE 3